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Abstract

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Recent measurements of nuclear level parameters are used to make more accurate calculations of the thermonuclear reaction rates for the following reactions:

1) $3He^4 \rightarrow C^{12}$

2) $C^{12} + He^4 \rightarrow O^{16} + \gamma$

3) $C^{13} + He^4 \rightarrow O^{16} + n$

4) $N^{14} + He^4 \nearrow F^{18} + \gamma$, $\searrow O^{17} + p$

5) $N^{15} + He^4 \rightarrow F^{19} + \gamma$

6) $O^{16} + He^4 \rightarrow Ne^{20} + \gamma$

The rates are calculated for temperatures up to $T = 50 \times 10^8$ °K.

AUTHOR

I. Introduction and Summary

Helium burning is an important stage in stellar evolution. Pure Helium burning ($3He^4 \rightarrow C^{12}$) and to a lesser extent the subsequent reactions $C^{12}(\alpha,\gamma)O^{16}$, and $O^{16}(\alpha,\gamma)Ne^{20}$ are expected to become the source of energy generation when the stellar core reaches temperatures of $T = 1 - 3 \times 10^8$ °K. These reactions also serve to synthesize C^{12}, O^{16} and possibly Ne^{20}, Mg^{24}, Si^{28}, etc.

Recent measurements of the decay widths of the second excited state of C^{12} now enable us to obtain an expression for the rate of $3He^4 \rightarrow C^{12}$ which is accurate to better than a factor two over the temperature range 0.85×10^8 °K to 10×10^8 °K. At higher temperatures the rate of pure Helium burning is still of considerable interest for the study of supernovae processes (Hoyle and Fowler, 1960). We can now estimate this high temperature rate with improved accuracy as a result of a recent determination of the spin of the third excited state of C^{12}.

Calculation of the thermonuclear reaction rates for the reactions $C^{12}(\alpha,\gamma)O^{16}$ and $O^{16}(\alpha,\gamma)Ne^{20}$ have been made previously. Salpeter (1957), Cameron (1959a,b,c), Reeves and Salpeter (1959), and Hayashi, Nishida, Ohyama and Tsuda (1959). More recent measurements have been made of level parameters for excited states in the nuclei O^{16} and Ne^{20} which narrow down existing uncertainties in the rates at $T > 2 \times 10^8$ ^oK. Such a procedure is at least in the right direction for obtaining a better estimate of the rate in the region $1 - 2 \times 10^8$ ^oK which depends on the properties of the excited states in the nuclei O^{16} and Ne^{20} hardly accessible to investigation by experiment.

The helium burning reaction $C^{13}(\alpha,n)O^{16}$ is of great importance. The rate for this reaction has been previously calculated for $T < 3\times10^8$ ^oK by several authors (Cameron 1955, 1957 and 1959a); Marion and Fowler (1957); and Salpeter (1955). Calculations which extend the rate to higher temperatures have been made by Reeves and Salpeter (1959) and Hayakawa, Hayashi, and Nishida (1960); however, the two results differed by a large factor.

In this paper we will give the results of a detailed study of the reaction $C^{13}(\alpha,n)O^{16}$ made by Thibaudeau (1962) over the broad range of temperature $0.2 \times 10^8 < T < 50 \times 10^8$ °K in which he has succeeded in narrowing down the uncertainty in the rate to about a factor 5. The most recent experimental data has been used.*

Rates have also been calculated over the same temperature range for reactions arising from the collisions of He^4 with N^{14} and N^{15};

At $T < 20 \times 10^8$ °K, the $N^{14} + He^4$ system can be resonant through the 4.651 state, and consequently the rate is 10^5 times larger than the rate quoted in Burbidge, Burbidge, Fowler and Hoyle (1957). The rate at $T > 20 \times 10^8$ °K obtained here for $N^{14} + He^4$ is probably accurate to better than a factor five. This reaction may act as a trigger for the Helium burning flash (Cameron 1959c).

In each case, we give the ratio of the reaction rate to the rate of the reaction $3He^4 \rightarrow C^{12}$ at the same temperature. This ratio is relatively insensitive to any screening effects.

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*A more recent analysis by Caughlan (private communication) has been incorporated here.

II Method of Calculation

For calculating the thermonuclear reaction rates we use the formalism developed by Salpeter (1957) as modified by Reeves and Salpeter (1959) for the higher temperatures considered.

The quantity which we are interested in obtaining is $p/\rho X_\alpha$ for which p is the probability per nucleus of type 2 of a reaction with nucleus of type 1 in a gas with density ρ and fractional concentration by mass of alpha particles: X_α The gas is assumed to be in equilibrium at temperature T which we will always measure in units of 10^8 °K.

The "resonant contribution" to p from a given level in the compound nucleus is determined by the integrated resonance strength:

$$\frac{\omega \Gamma_\alpha \Gamma_{out}}{\Gamma} \approx \omega \left\{ \text{Smaller of } \Gamma_\alpha \text{ or } \Gamma_{out} \right\} \tag{1}$$

In this expression, Γout is the particle width of the exit channel and must be considered separately depending on whether the emitted particle is a photon, proton or neutron. Γ_α is the alpha particle width at resonance. If we denote by E_r the alpha particle resonance energy then we can write

$$\Gamma_\alpha = \gamma \exp(2\pi\eta_r + dE_r) \tag{2}$$

where

$$2\pi\eta_r = 0.989\, Z_1 Z_2\, (M/E_r)^{1/2} \tag{3}$$

and

$$d = 0.122 \left(\frac{MR^3}{Z_1 Z_2}\right)^{1/2}$$

In equation (2), γ is a constant which includes the Coulomb and orbital angular momentum barrier penetration effects but is almost independent of E_r. This approximation to Γ_α is expected to be valid when E_r is much less than the Coulomb barrier. For all of the reactions studied in this paper this condition is well satisfied. In particular at the highest temperatures ($T \sim 50 \times 10^8$ °K) where E_r becomes comparable with the Coulomb barrier it was not necessary to use approximation (2) because either an experimental value of Γ_α was available or $\Gamma_\alpha > \Gamma_{out}$.

At the lower range of temperature ($T << 50 \times 10^8$ °K), there is in addition a "non resonant contribution" to p which is completely determined by the low energy cross section factor S'. We write this as

$$S'_{BW} = \frac{N_{nr}}{(E'_m(T) - E_r)^2 + \Gamma^2/4} \tag{4}$$

In many cases the term $\Gamma^2/4$ can be neglected. The denominator is the familiar Breit-Wigner resonance denominator corresponding to an optimum bombarding energy at temperature T.

The numerator is temperature independent and may be written as

$$N_{nr} \approx \frac{0.647}{M}\,\omega\gamma\,\Gamma_{out} \tag{5}$$

For those levels in which the non resonant contribution was dominant, it was necessary either to extract γ from measured values of the alpha width at resonance using equation (2) or to estimate γ theoretically. For this purpose we write it as

$$\gamma = \xi_{\ell}'^{2}\,\gamma^{*} \tag{6}$$

The first factor $\xi_{\ell}'^{2}$ is of the form

$$\xi_{\ell}'^{2} = (G_0^{2}/G_{\ell}^{2})\,2g\,e^{4g} \tag{7a}$$

where $g = 0.2635\,[Z_1 Z_2 MR]^{1/2}$

The values of G_0^2/G_{ℓ}^2 in which G_{ℓ} is the irregular Coulomb wave function for orbital quantum number ℓ may be obtained with the help of recursion relations given in Feshbach, Shapiro and Weisskopf (1953) or with the aid of graphs of the Coulomb functions given by Sharp, Gove and Paul (1955).

The second factor γ^{*} is the reduced width of the level in energy units. We can write it as

$$\gamma^{*} = \theta_{\alpha}^{2}\left(\frac{3\hbar}{2MR^{2}}\right) \tag{7b}$$

in which $0 < \theta_\alpha^2 < 1$. Usually it is possible to estimate θ_α^2 from available measurements of Γ_α for neighboring levels.

III Nuclear Reaction Rates

For each reaction we first illustrate the situation by plotting the position and width of the Gamow peak at various temperatures on the diagram of excited levels in the compound nucleus. Such diagrams show immediately which levels will contribute significantly to the rate at any temperature. We then give tables and a discussion of the nuclear level parameters, pointing out in particular the improvements in our knowledge of these parameters. For each level which might contribute to the nonresonant rate, we tabulate values of Υ and S_{BW}. Such a procedure will allow rapid recalculation of the rates if new data becomes available in the future.

We have used the most recent set of Q values and mass differences (Everling, Koenig, Mattauch and Wapstra 1960).

(a) Rate of the $3\alpha \rightarrow C^{12}$ reaction.

The situation is descrdbed in Fig. 1, and Fig. 2. For the resonance energy E_r ($E_r = E^* - 3\alpha$) of the second excited state in C^{12*}, we use E_r = 375 Kev. This value is chosen to be in the range 372 ± 4 Kev determined by the experiment of Cook, Fowler, Lauritsen and Lauritsen (1957); at the same time it gives an excitation of 7.650 Mev for the second excited state of C^{12} which is within the range 7.656 ± 0.007 Mev quoted by Ajzenberg-Selove and Lauritsen (1959).

A large amount of theoretical and experimental work has been done to establish the nuclear parameters of the 7.65 Mev state.

The spin and parity of the 7.65 Mev state of C^{12} are almost certainty 0+ (Cook, Fowler, Lauritsen and Lauritsen 1957; Alburger 1960; Ajzenberg-Selove and Stelson 1960). This means that in addition to a possible alpha decay back to $\alpha + Be^8$ from whence this level was formed, a transition to the 0+ ground state can occur either by emission of an electron-positron pair or by a gamma ray cascade through the 2+ first excited state consisting of two electric quadrupole transitions in sequence. The rate of the $3\alpha \rightarrow C^{12}$ reaction is then proportional to the quantity:

$$\Gamma_{Rad} = \Gamma_{e^\pm} + \Gamma_\gamma \tag{8}$$

where $\Gamma_{e^\pm}$ and Γ_γ are respectively the widths for pair and gamma decay. We will show that $\Gamma_\gamma / \Gamma_{e^\pm} \approx 50$ so that

$$\Gamma_{Rad} \approx \Gamma_\gamma \tag{9}$$

Since no direct experimental determination of Γ_γ has yet been made, it must be deduced indirectly from measurements of ratios of the widths.

The ratio of the pair width to the total width $(\Gamma = \Gamma_{Rad} + \Gamma_\alpha)$ is determined by a combination of two experiments (Alburger 1960; Ajzenberg-Selove and Stelson 1960) These yield

$$\Gamma_{e^\pm}/\Gamma = (6.6 \pm 2.2) \times 10^{-6} \tag{10}$$

The ratio of the gamma width to the total width has been measured by two independent experiments. Alburger (1961) gives:

$$\Gamma_\gamma/\Gamma = (3.3 \pm 0.9) \times 10^{-4} \tag{11}$$

A more precise value has recently been obtained (Seeger 1963; Seeger and Karanagh 1963):

$$\Gamma_{Rad}/\Gamma = (2.82 \pm 0.29) \times 10^{-4} \tag{12}$$

We will adopt

$$\Gamma_\gamma/\Gamma = (3.0 \pm 0.3) \times 10^{-4} \tag{13}$$

All of the experimental information is then embodied in the single ratio

$$\Gamma_\gamma/\Gamma_{e^\pm} = 45 \pm 16 \tag{14}$$

According to the theory of Oppenheimer and Schwinger (1939) and Dalitz (1951), the width for pair emission in a $0+ \rightarrow 0+$ transition can be related to a matrix element (M.E.) for this process by:

$$\Gamma_{e^\pm} = \left(\frac{e^2}{\hbar c}\right)^2 \frac{K^5(\hbar c)}{135\pi} |M.E.|^2 \tag{15}$$

where $K = \gamma/\hbar c$

and γ is the energy released in the transition. The value of M.E. can be extracted from measured cross sections for elastic and inelastic scattering of electrons leading to the 7.65 Mev state (Schiff 1954, 1955).

From the electron scattering experiments of Fregeau and Hofstader (1955), Schiff obtained the value

$$M.E. = 3.8 \times 10^{-26}\ cm^2 \tag{16}$$

Later Fregeau (1956) carried out the same type of analysis on his own data and got

$$M.E. = (5 \pm 1.2) \times 10^{-26}\ cm^2 \tag{17}$$

Walecka (1962) has analyzed the data of Fregeau and Hofstader using a model in which the C^{12} nucleus is explicitly represented by an oscillating incompressible liquid drop. This model predicts a value

$$M.E. = 4.2 \times 10^{-26}\ cm^2 \tag{18}$$

this value, being model dependent, is less reliable than (17). With Fregeaus M.E. We get

$$\Gamma_{e^\pm} = (5.5 \pm 3) \times 10^{-5}\ ev. \qquad \text{and from (7)} \quad (19)$$

$$\Gamma_\gamma = (2.5 \pm 1.6) \times 10^{-3}\ ev. \tag{20}$$

This value of the 3.23 E2 gamma radiation width for C^{12}

is close to a theoretical single particle estimate made by Ferrell (1957). His result was $\Gamma_\gamma = 1.4 \times 10^{-3}$ ev with an uncertainty of a factor two.

The total width may be obtained from either (10) and (19) or from (13) and (20)

$$\Gamma = 8.3 \pm 5.3 \text{ ev.} \tag{21}$$

We can also obtain an estimate of Γ independent of any uncertainty in Γ_γ or $\Gamma_{e^\pm}$, since by equations (10) and (13) we have

$$\Gamma \approx \Gamma_\alpha \tag{22}$$

An upper limit to the alpha particle width Γ_α is given by the so called Wigner limit (Blatt and Weisskopf 1952). For s-wave alpha particles the Wigner limit depends only on the assumed interaction radius R for the $\alpha + Be^8$ system. Taking the usual recipe $R = r_0 (A_1^{1/3} + A_2^{1/3})$ fe. We find that a choice $r_0 = 1.5 \pm 0.2$ or R = 5.4 $\pm$ 0.7 gives

$$\Gamma_\alpha \leq 7^{+7}_{-3} \tag{23}$$

Equations (21), (22) and (23) indicate that Γ_α is equal to the Wigner limit and that R $\approx$ 5.4.

There is now strong evidence for an assignment of

$J\pi = 3^-$ for the 9.63 Mev level of C^{12} (Bradford 1961 and Carlson 1961). We know also that the alpha width has the value $\Gamma_\alpha = 30 \pm 8$ kev (Douglass 1956); however, the radiation width is unknown. The most likely mode of radiative decay would be by cascade through the 2+ first excited state. If the width had an average value for uninhibited E1 transitions $\Gamma_\gamma = 2.5$ ev (Carlson 1961); however, this transition would be of the form 0 → 0 in isotopic spin, so that selection rules will probably reduce the width by an order of magnitude. A calculation of Γ_γ has been quoted (Hoyle and Fowler 1960) which shows that an admixture of 4% of the T = 1 level at 17.63 Mev will reduce Γ_γ to 0.01 ev. We choose $\Gamma_\gamma = 0.03$ ev with an uncertainty of about a factor 10 either way.

We have summarized the nuclear parameters for the reaction $\alpha + Be^8 \rightarrow C^{12*} + \gamma$ in table I.

Methods for obtaining the resonant rate have been discussed by Salpeter (1957). The mean rate of destruction of He^4 per alpha particle per second (denoted by p_α) is given by:

$$p_\alpha / (\rho x_\alpha)^2 = \omega \, \frac{2.37 \times 10^{-4}}{T^3} \left(\frac{\Gamma_{3.2\gamma} + \Gamma_{te}}{1 \text{ e.v.}} \right) \times 10^{-50.4\, E_r/T_9} \sec^{-1} \left(\frac{gm}{cm^3}\right)^{-2} \tag{24}$$

For the 7.65 Mev level, $\omega = 1$ and the new experimental values give

$$P_\alpha/(\rho x_\alpha)^2 = \frac{5.92 \times 10^{-7}}{T_8^3} \times 10^{-18.9/T_8} \tag{25}$$

Combining the uncertainty in $\Gamma_{3.2\gamma}$ with the uncertainty in E_r, we find that this resonance rate should be correct within a factor two all through the range $1 < T_8 < 50$. For the 9.63 level we get in a similar fashion

$$P_\alpha/(\rho x_\alpha)^2 = \frac{5 \times 10^{-5}}{T_8^3} \times 10^{-119.0/T_8} \tag{26}$$

with a possible error of a factor of ten either way.

The relative importance of the nonresonant rate is measured by the ratio $\wp_{nr}/\wp_r$ where $\wp_{nr}$ and $\wp_r$ are respectively the rates per pair of collisions between α and Be^8. Using an alpha width for each level of the form $\Gamma_\alpha(E) = \gamma \times 10^{(-5.612/\sqrt{E} - 0.371E)}$ we get the values given in table II. It is found that the nonresonant rate from both levels is negligible for $0.85 < T < 50$. Below $T \approx 0.7$ the nonresonant rate from the 7.65 level dominates the rate. The resultant rate for $3\alpha \rightarrow C^{12}$ is tabulated as $p_\alpha/(\rho x_\alpha)^2$ without screening in table III. The actual rate with corrections for electron screening included is plotted as $p_{\tilde{\alpha}}$ at various densities of He^4 in Fig.3.

Energy Production

Each reaction $3\alpha \rightarrow C^{12}$ releases an amount $Q = 7.275$ Mev of energy. The rate of energy generation is

$$e = 5.84 \times 10^{17} p_\alpha x_\alpha$$

In the range $0.85 < T_8 < 50$

$$\epsilon = \frac{3.46 \times 10^{21}}{T_8^3} x_\alpha^3 (\rho/10^5)^2 \times 10^{-18.9/T_8} \text{ erg/gm.sec.} \tag{27}$$

At $T \lesssim 0.7$

$$\epsilon = x_\alpha^3 (\rho/10^5)^2 S' \times 10^{28.81} \times 10^{-4.74/T_8} T_8^{-13/6} \times 10^{-22.05/T_8^{1/3}} \text{ erg/gm.sec.} \tag{28}$$

$$S' = 2.4 \times 10^{-4} / [0.15 T_8^{2/3} - 0.28]$$

where S' is given for the 7.65 level in table II.

Salpeter has defined a useful parameter n by the equation

$$\epsilon = \epsilon_0 (T/T_0)^n \quad \text{when } T \text{ is near } T_0.$$

We find for $0.85 < T_8 < 50$

$$n = \frac{43.5}{T_0} - 3 \tag{29}$$

For $0.6 < T_8 < 0.85$, the value of n must be computed using for θ the sum of the resonant and nonresonant rates as given by equations (27) and (28), respectively.

The preceding equations do not contain corrections for electron screening. To include this, we have added a scale to Fig. 3 from which ϵ/x_α may be read directly for various assumptions of ρx_α . In table V we have tabulated n and included the modification to equation (29) due to screening at various densities.

b) Rate of the reaction $C^{12}(\alpha,\gamma)\ O^{16}$

As can be seen from Fig 4, there are three levels which give the main contribution to the rate. (See also table VI).

The 7.12 Mev level lies below the sum of the masses of $C^{12} + \alpha$ and makes a nonresonant contribution which dominates the rate at $T_8 < 10$. We use the recently measured value $\Gamma_\gamma = 6.6 \times 10^{-2}$ ev with an uncertainty of about 30% (Swann and Metzger 1957; Reibel and Mann 1960). The alpha width can be written as $\Gamma_\alpha(E) = \gamma e^{-20.53/E^{1/2} - 0.739E}$ with $\gamma = \xi_1'^2 \theta_\alpha^2 \gamma_w$. The factor $\xi_1'^2$ depends mainly on the radius R. Following Salpeter we take R = 5.27 and get $\xi_1'^2 = 4.9 \times 10^6$; the uncertainty in R is not more than 10%, which introduces an uncertainty of a factor of about two in $\xi_1'^2$. The factor γ_w is the Wigner upper limit to the reduced width in energy units, $\gamma_w = 0.74$ Mev. Finally θ_α^2 is an unknown nuclear parameter which measures the overlap of the 7.12 Mev state with the system $C^{12} + \alpha$. This quantity may be estimated statistically from the

spectrum of values of θ_α^2 for the levels at slightly higher energy. From a table given by Roth and Wildermuth (1960) we find that θ_α^2 is distributed between 0.001 and 1 for all levels with $E^* \leq$ 12.5. There is some theoretical reason (quoted in Cameron 1958) to believe that this state should have a rather large alpha particle width. We shall take $\theta_\alpha^2 = 0.1$ with an error of about a factor ten either way. This may be compared with Salpeter's choice of $\theta_\alpha^2 = 0.07$.*
The nonresonant rate is given by

$$p_{nr}/\rho x_\alpha = S' \times \frac{1.15 \times 10^{10} \times 10^{-30.05/T_8^{1/3} - \delta}}{T_8^{2/3}} \tag{30}$$

where $\delta = 0.06\, T_8^{2/3}\,(1 - 0.067\, T_8^{1/3})$

With our values we get

$$p_{n.r.}/\rho x_\alpha = \frac{4.6 \times 10^{9} \times 10^{-30.05/T_8^{1/3} - \delta}}{T_8^{2}[1 - 4 \times 10^{-3} T_8 + 0.2\, T_8^{-2/3}]^2} \ \sec^{-1} (gm/cm^3)^{-1} \tag{31}$$

This expression is an approximation to the exact rate which is

$$p/\rho x_\alpha = 1.7 \times 10^{11} T^{-3/2} \sec^{-1} (gm/cm^3)^{-1} \int_0^\infty dE\ E e^{-E/kT} \sigma(E) \tag{32}$$

In order to test the accuracy of the nonresonant approximation, we have carried out a hand integration at $T_9 = 1$. The cross section was taken to be of the single

*Some astrophysical arguments seem to favor the value $\theta_\alpha^2 \simeq 1$ Since P is proportional to θ_α^2, its value, and the value of q_{16} defined later, would simply be increased by 10. However, we still find it safer to use the present lower value.

level Breit-Wigner form

$$\sigma(E) = (2J+1)\,\frac{\pi \lambda\!\!\!^{-2}\,\Gamma_\alpha(E)\,\Gamma_\gamma}{(E+0.041)^2+\Gamma^2/4}$$

with

$$\Gamma_\alpha(E) = 3.7\times 10^{5}\, e^{-20.53/E^{1/2} - 0.739E}$$

We find that both methods are in agreement and give

$p/\rho x_\alpha = 3.3\times 10^{-21}$ at $T_8 = 1$. This is within a factor two of Salpeter (1957). Our rate is still uncertain by a factor of twenty.

For the broad level at 9.58, much better accuracy is possible because the factor γ can be determined from the known value at resonance $\Gamma_\alpha = 0.65$ Mev (Segel, Olness and Sprenkel 1961; Miller, Phillips, Harris, Beckner 1962). For the radiation width, we use the measured value Γ_γ = 6×10^{-3} ev within a factor of about two (Bloom, Toppel, and Wilkinson 1957). The nonresonant rate is again expressed by equation (30) with S' from table VI; the resonant rate can be written as

$$p_r/\rho x_\alpha = \frac{4.3\times 10^{3}\times 10^{-122/T_8}}{T_8^{3/2}} \qquad (33)$$

The 9.58 level does not contribute below $T_8 \approx 10$ but for $10 \lesssim T \lesssim 20$ it gives a rate which is comparable with the 7.12 level. The uncertainty in this range is probably somewhat less than a factor ten.

Above $T \approx 30$ resonant capture into the level at 9.84 tends to dominate the rate. The alpha width has been measured by Hill (1953), and the radiation width by Meads and McIldowie (1960). Since $\Gamma_\gamma < \Gamma_\alpha$ and $\Gamma_\gamma = 0.02 \pm 0.01$ ev, the rate is uncertain by less than a factor two either way. We get

$$p/\rho x_\alpha = \frac{2.35 \times 10^{-135/T_9}}{T_9^{3/2}} \tag{34}$$

The resultant rate is tabulated without screening as $p/\rho x_\alpha$ in table VII.

The energy production for $C^{12}(\alpha,\gamma)O^{16}$ is related to p by $\epsilon = 5.76 \times 10^{17}\ p x_C$.

Because of the large uncertainty in θ_α^2, there is need for a measurement of the low energy cross section factor S' for the $C^{12}(\alpha,\gamma)O^{16}$ reaction+. The experiment of Allan and Sarma (1955) is not very helpful in this respect; their results imply only that $S' < 10^5$ Mev-barns.

c) Reaction rate for $O^{16}(\alpha,\gamma)Ne^{20}$

The situation is described in table VIII and Fig. 5.

+ A study of this reaction is now under way at the Kellog Radiation Laboratory (J. D. Larson-private communication).

We have incorporated in these the new data from the Chalk River group (Kuehner, Gove, Litherland, Clark, and Almqvist 1961). The Q value and the various resonance energies are known within about 10 Kev.

The 4.97 level is a (2 -); hence not active for O^{16} (α,γ) Ne^{20}.

For the 5.64 Mev (3-) level, we have two independent measurements:

$$(2\ell+1)\frac{\Gamma_\alpha \Gamma_\gamma}{\Gamma} = 0.003 \pm 0.002 \text{ ev} \qquad \text{and}$$

$$\Gamma_\alpha / \Gamma = 0.07 \pm 0.01$$

From these we get

$$\Gamma_\alpha = 6 \pm 4 \quad \times 10^{-3} \text{ ev}$$
$$\Gamma_\gamma = 4 \pm 3 \quad \times 10^{-4} \text{ ev}$$

For the 5.80 Mev (1 -) level we have only one piece of information

$$\Gamma_\gamma / \Gamma < 0.006$$

The single particle limit for Γ_α is 16 ev; the actual Γ_α has been estimated to be about 10% of this value, or $\Gamma_\alpha \approx 2$ ev.

We adopt this value and from the experimental information $\Gamma_\gamma/\Gamma_\alpha < .006$, we choose $\Gamma_\gamma = 0.01$ ev. Both choices should be valid within factors of 10 either way. The product $(2l + 1)\frac{\Gamma_\alpha \Gamma_\gamma}{\Gamma}$ should be about 0.03 ev again within factors of 10.

For the 6.75 Mev level the radiation width is not known. On the basis of single particle estimates we take $\Gamma_\gamma = 0.1$ ev. The alpha width is 19 Kev (Cameron 1953).

Calculation of nonresonant rates again involves a knowledge of the S' factor belonging to the various resonances, and for that a knowledge of the factor $\underline{\gamma}$. The values of these quantities are given in table VIII. The levels at 6.75 and 5.80 are seen to be the most important contributors. Higher levels would make negligible small additions which would still be within the experimental uncertainties.

The nonresonant rate is given by

$$p/\rho x_\alpha = 1.5 \times 10^{10}\, S' T_8^{-2/3} e^{-35.6/T_8^{1/3}} \sec^{-1} (gm/cm^3)^{-1} \qquad (35)$$

In the range $1 < T_8 < 2$, the S' factors are roughly constant; their average values are given in table VIII. We get

$S' \approx 0.3$ Mev barns and

$$P/\rho x_\alpha = 4.5 \times 10^9\, T_8^{-2/3} \exp(-85.6/T_8^{1/3})\ \mathrm{sec}^{-1} (\mathrm{gm/cm^3})^{-1}. \tag{36}$$

both within factors of 10. This rate is 4.5 times as large as the estimate of the nonresonant rate made by Salpeter; mainly because we know more about the properties of the levels.

The resonant rate is given by

$$P/(\rho x_\alpha) = 2.1 \times 10^{11} (\omega \Gamma_\alpha \Gamma_\gamma/\Gamma)\left[10^{-50.4 E_r/T_8}\right] T_8^{-3/2}. \tag{37}$$

The resonance at 5.64 dominates the rate for $2<T_8<8$; in this region we know the rate within a factor 2.*

From T = 8 to T = 20, the 5.64 and 5.8 rates are roughly similar (uncertainty of about a factor ten).

Above T = 20 we include contributions from the 6.75 and higher levels. A statistical study has been made which yields the numbers given. The accuracy should be better than a factor of ten.

*Edison and Bent (1962) find evidence for a somewhat larger value of Γ_γ/Γ for the 5.64 level; they obtain $\Gamma_\gamma/\Gamma = 0.3 \pm 0.2$. This result does not affect our rate which depends only on the value of $(2\ell + 1)\frac{\Gamma_\alpha \Gamma_\gamma}{\Gamma}$.

The resultant rate is tabulated as $p/\rho x_\alpha$ in table IX, without screening.

The energy production is

$$\epsilon = 2.8 \times 10^{17}\, p x_0 \text{ erg/gm-sec} \tag{38}$$

We will now describe the results of a detailed study of the four reactions [$C^{13}(\alpha, n)O^{16}$, $N^{14}(\alpha, \gamma)F^{18}$, $N^{14}(\alpha, p)O^{17}$, and $N^{15}(\alpha, \gamma)F^{19}$] made by Thibaudeau (1962). In each of these reactions the compound nucleus possesses a large number of excited states in the region of energy which falls within the Gamow peak at the temperatures of interest. The situation is illustrated in Figures 6, 7 and 8. Under such conditions the total thermonuclear reaction rate is correctly obtained by summing up the resonant and nonresonant rates from each level near or within the Gamow peak. For each of the four reactions the unscreened rate can be symbolically written as

$$P/\rho x_\alpha = \frac{2.2 \times 10^{11}}{T_8^{3/2}} \sum_r \left(\omega \frac{\Gamma_\alpha \Gamma_{out}}{\Gamma}\right) 10^{-50.4 E_r/T_8}$$
$$+ \frac{1.5 \times 10^{10}}{T'^{2/3}} \times 10^{-c/T'} \sum_r N_{nr} \Big/ \left((E_m' - E_r)^2 + \Gamma_r^2/4\right). \tag{39}$$

where T' is the modified temperature as defined by Reeves and Salpeter and C = 30.25, 33.71 and 33.88 for $C^{13}+\alpha$, $N^{14}+\alpha$ and $N^{15}+\alpha$ respectively,

d) Rate of the reaction $C^{13}(\alpha,n)\ O^{16}$

The positions, widths and spins of the levels occuring in the sums of equation (39) (see Fig. 9) were obtained from Ajzenberg and Lauritzen (1961); the most recent measurements of Fossan, Walter, Wilson and Barschall (1961) have brought only minor changes to these values and were used whenever their results were not even more uncertain than those of Ajzenberg and Lauritzen.

The Γ_α are provided by an experimental study of Walton, Clement and Boreli (1957). These authors have determined the ratio Γ_n/Γ_α for a dozen levels; the value of Γ_α is obtained by setting $\Gamma=\Gamma_n+\Gamma_\alpha$. Furthermore they have calculated the θ_α^2 by using a radius of R = 5.7 fe. The values obtained range between 0.1 and 0.004 and are known to better than a factor 2 except those which are known to only an order of magnitude. Two sets of the θ_α^2 have been considered corresponding to two possible values of ℓ since the parity of these levels is not known; we have reproduced only one set. For all the other levels we take θ_α^2 = 0.02 within a factor 5.

In cases where the spin and parity of the levels are unknown, it is necessary to set $\omega = 1$. The only important case occurs for a level at an excitation of 6.869 Mev. We have taken $\ell = 1$ and $\omega = 1$ corresponding to $J\pi = \frac{1}{2}+$. Therefore the contribution from this level can hardly increase more than a factor 10, and if it decreases by more than a factor 10, the contribution from the other levels will predominate. In this way we minimize the uncertainty in the rate of this reaction for temperatures $3 \leq T_8 \leq 5$.

We have assumed that $\Gamma_n \gg \Gamma_\alpha$ in all cases and set

$$\omega \frac{\Gamma_\alpha \Gamma_n}{\Gamma} = \omega \Gamma_\alpha$$

unless some evidence indicated the contrary.

For the nonresonant contribution however it is necessary to know Γ_n. If the value is unknown we choose $\Gamma_n = 1$ Kev, which is just a little less than the smallest measured width; the results are not especially sensitive to this choice. Only the levels at 6.37 and 7.28 make an appreciable nonresonant contribution.

In this way we find that only one or a few of the large number of levels occuring in the sum of equation 3 provide the principal contribution to the rate at any temperature and determine also the uncertainty in the rate. We will refer to these as "key levels." The parameters for these key levels are given in table X. The rate (unscreened) of the reaction C^{13} (α, n) O^{16} is given in table XI with the source of the principal contribution.*

The rate of production of energy is

$$\mathcal{E} = 1.6 \times 10^{17} \rho X_c \quad (\text{ergs/gm-sec}) \tag{40}$$

where X_c is the relative concentration by mass of C^{13}.

e) Rate of the reaction $N^{14} + \alpha$

The parameters of the levels in F^{18} in the region of excitation in which we are interested (see fig. 7) are even less well known than for O^{17}. Furthermore, there are two possible reactions: N^{14} (α, γ) F^{18} and N^{14} (α, p) O^{17}; the latter dominates for temperatures $T_8 \geq 20$.

Values of Γ_α were taken from the measurements of Herring (1958), Silverstein, Salisbury, Hardie and Oppliger (1961) and Brown (1962). However, while Herring as well as Silverstein et. al. use a radius of 4.8 fermi to determine θ_α^2, Brown takes R = 5.6 fe which is probably more reasonable. We have recalculated all the θ_α^2 with R = 5.6 fermis (and restored to it a factor 2/3 which

*Note

The nonresonant rate has been recently reconsidered by Caughlan (private communication), She has obtained an estimate of the S value by an analysis of the background (off-resonance) experimental values. She obtains:

$$S = 10^{6}\left[10.6 - 9.15\ E(\mathrm{Mev})\right] \quad \text{Mev barns}$$

from there we obtain:

$$\log p/\rho x_{\alpha} = 16.2 + \log(10.6 - 1.2T_8^{2/3}) - \frac{30.24}{T_8^{1/3}} - 0.08T_8^{2/3}$$

Her reults are used in Table XI.

Brown dropped in his definition of θ_α^2). The values thus obtained fall between 0.013 and 0.32, and for all the other levels we choose $\theta_\alpha^2 = 0.06$ within a factor 5 and R = 5,6 fermis.

In order to complete the calculation of Γ_α, when the spin J of the level is known, we have assumed that the total contribution to the reaction was due to alpha particles with the smallest orbital momentum compatible with the data. When J is unknown, we have assumed that it can vary from 0 to 4 and ℓ from 0 to 3, and we have taken for the value of $\omega\Gamma_\alpha$ the geometrical mean of the values obtained by making $\ell = 0$ and $\ell = 3$ with an uncertainty equal to the ratio between the mean value and the extreme value.

The value of Γ_γ is known only for four levels (Price 1955, Phillips 1958). These values go to 6 ev. Because of the large number of levels below those in which we are interested, we believe that an E1 transition will be possible in every case. We choose in every case $\Gamma_\gamma = 3$ ev within a factor 3.

The nonresonant contribution predominates for $T_8 < 0.5$. For the reaction $N^{14}(\alpha,p)O^{17}$ we know only two values of Γ_p (Brown 1962) and the corresponding θ_p^2 calculated by taking R = 5.0 fermis; these are $\theta_p^2 = 0.014$ and 0.044 respectively. On the other hand Lane (1960) gives in F^{17} the values of θ_p^2 for four levels, $0.01 \le \theta_p^2 \le 0.16$.

Therefore, we take $\theta_p^2 = 0.04$ to within a factor 4 and we calculate Γ_p by the method indicated previously for Γ_α. When Γ is known, $\Gamma_\alpha \Gamma_p / \Gamma$ cannot exceed $\frac{\Gamma}{4}$ and this is taken account of in the evaluation. The nonresonant contribution to $N^{14}(\alpha,p)O^{17}$ is negligible. It is removed from the Gamow peak at low temperatures by conservation of energy, and furthermore the emission of protons is hindered by the Coulomb barrier.

Again we find that only certain key levels are important in determining the rate at each temperature with its uncertainty. The parameters for these key levels are given in table XII and the total rate for $N^{14} + \alpha$ (unscreened) is given in table XIII.

If one designates by X_n the relative concentration by mass of N^{14}, the rate of production of energy is

$$\mathcal{E} = 4.2 \times 10^{17} \rho X_n \quad (ergs/gm\text{-}sec.)$$

for T<20($N^{14} + \alpha \rightarrow F^{18} + \gamma \rightarrow O^{18} + e^+ + \nu + \gamma$; 6.08 Mev per reaction). For $T \gtrsim 20$, the reaction is endothermic: $(N^{14}+\alpha \rightarrow O^{17}+p)$ dominates.

$$\mathcal{E} = -8.2 \times 10^{16} \rho X_n \ (ergs/gm\text{-}sec)$$

f) Rate of the reaction $N^{15}(\alpha,\gamma)\ F^{19}$

The positions of the levels have been taken from general references (Ajzenberg-Selove and Lauritsen 1961); (Way et. al. 1961), except for several most recent corrections which come from measurements made by Silbert and Jarmie (1961). (See fig. 8)

The experiments of Smotrich, Jones, McDermott and Benenson (1961) on the elastic scattering of alpha particles by N^{15} have determined the widths of several levels to a precision around 10%. The values of θ_α^2 which they obtain range between 0.005 and 0.4. We take for the other levels θ_α^2 = 0.05 within a factor 8.

We treat the problem of unknown spins and orbital momenta as for the reaction $N^{14}(\alpha,\gamma)\ F^{18}$.

Price (1957) has established the value of $\omega\Gamma_\gamma$ for three levels. The corresponding Γ_γ are compressed between 2 ev and 7 ev; we choose Γ_γ = 4 ev for the other levels.

In table XIV we give parameters for the key levels in F^{19}; the resulting rate is given in table XV.

Reeves and Salpeter indicate the same rate for

$N^{14}(\alpha,\gamma)\,F^{18}$ and $N^{15}(\alpha,\gamma)\,F^{19}$; our rate for $N^{15}(\alpha,\gamma)\,F^{19}$ deviates rarely more than a factor 10.

If one designates by X_n the relative concentration by mass of N^{15}, the rate of production of energy is ($Q = 4.01$ Mev).

$$\mathcal{E} = 2.6 \times 10^{17} \rho X_n \ (\text{ergs/gm-sec})$$

Finally we give as one main result of our work, graphs (see figure 9,) of certain ratios which occur in equations determining the rate of formation of the isotopes C^{12}, O^{16} and Ne^{20} in the Helium thermonuclear reactions. The ratios plotted are defined by:

$$\log q_{12,\alpha}\rho = \log p/\rho x_\alpha \ (\text{for } C^{12}(\alpha,\gamma)O^{16}) - \log p/(\rho x_\alpha)^2 \ (\text{for } 3\alpha \to C^{12})$$

$$\log q_{16,\alpha}\rho = \log p/\rho x_\alpha \ (\text{for } O^{16}(\alpha,\gamma)Ne^{20}) - \log p/(\rho x_\alpha)^2 \ (\text{for } 3\alpha \to C^{12})$$

TABLES

TABLE II - Nonresonant Rate for $3\alpha \rightarrow C^{12}$

Level	7.65	9.63
χ (Mev)	3.9×10^5	1.1×10^3
S' (Mev barns)	$\frac{2.4 \times 10^{-4}}{(E'_m - 0.28)^2}$ $E'_m = 0.15\, T_6^{2/3}$	$\frac{8.0 \times 10^{-6}}{(E'_m - 2.26)^2}$ $E'_m = 0.15\, T_6^{2/3}$
ρ_{nr}/ρ_r	$S' \times 10^{7.27} T^{6/6} \times 10^{(14.16/T - 22.05/T^{1/3} - \delta)}$	$S' \times 10^{5.35} T^{5/6} \times 10^{\left(\frac{113.9}{T} - \frac{22.05}{T^{1/3}} - \delta\right)}$
	where $\delta \equiv 0.022 T^{2/3} (1 - 0.21\, T^{1/3})$	

TABLE III

Values of $p_\alpha/(\rho x_\alpha)^2$ for $3\alpha \longrightarrow C^{12}$

T	$\log p_\alpha/(\rho x_\alpha)^2$	
0.60	-34.67	7.65 Nonresonant (uncertainty $\sim$4)
0.70	-32.17	
0.85	-28.29	
1.0	-25.13	
1.2	-22.22	
1.4	-20.17	
1.6	-18.65	
1.8	-17.50	7.65 Resonant (uncertainty $\sim$ 2)
2	-16.58	
3	-13.96	
4	-12.76	
5	-12.11	
6	-11.71	
8	-11.30	
10	-11.12	
20	-11.07	
30	-11.29	7.65 and 9.63 Resonant (uncertainty
40	-11.50	$\sim$ 10)
50	-11.50	

TABLE IV

Estimated corrections to log p due to electron screening.

T \ ρ	$3\alpha \rightarrow C^{12}$ 10^4	10^5	10^6	10^7	10^8	$C^{12}(\alpha,\gamma)O^{16}$ 10^4	10^5	10^6	10^7	10^8	$O^{16}(\alpha,\gamma)Ne^{20}$ 10^4	10^5	10^6	10^7	10^8
0.6	0.3	0.5													
0.7	0.2	0.4													
0.85	0.2	0.4	0.8												
1.0	0.1	0.3	0.7	1.6		0.1	0.3	0.6	1.4		0.1	0.4	0.9	1.7	
1.2		0.3	0.6	1.3			0.3	0.5	1.1			0.3	0.7	1.4	
1.4		0.2	0.5	1.1			0.2	0.5	1.0			0.3	0.6	1.2	
1.6		0.2	0.5	1.0			0.2	0.4	0.9			0.2	0.5	1.1	
1.8			0.4	0.9				0.4	0.8			0.2	0.5	1.0	
2.0			0.4	0.8				0.3	0.7			0.2	0.4	0.9	
2.4			0.3	0.6				0.3	0.6				0.3	0.7	
3			0.2	0.5				0.2	0.5				0.3	0.6	
3.5				0.4					0.4				0.2	0.5	
4				0.4					0.3					0.4	
5				0.3					0.3					0.3	
6				0.2					0.2					0.3	
7				0.2										0.2	
10					0.3					0.2					0.3
20					0.1					0.1					0.1

Corrections to log p due to the effects of electron screening were calculated by the method of Salpeter (1954). Weak screening is used for $\Delta \log p \lesssim 0.3$. For larger values the strong screening approximation was used and compared with the numerical method valid for arbitrary screening strength; the difference in these methods is always less than a factor two. The table has been prepared assuming $x_\alpha \approx 1$.

TABLE V

Values of n (the energy production exponent) for $3\alpha \longrightarrow C^{12}$ at various temperatures and densities

	10^2	10^3	10^4	10^5	10^6	10^7	10^8
	37.0	36.7	36.2	35.7			
0.[illegible]5	47.4	47.2	46.9	46.5	45.4		
1.0	40.5	40.4	40.1	39.3	38.8	36.9	
1.2	33.2	33.2	32.9	32.3	31.8	30.2	
1.4	28.1	28.1	27.9	27.4	26.9	25.5	
1.6	24.2	24.2	24.2	23.6	23.1	21.9	
1.8	21.2	21.2	21.2	20.7	20.3	19.2	
2.0	18.8	18.8	18.8	18.4	17.6	17.0	
3	11.5	11.5	11.5	11.3	10.9	10.3	
4	7.9	7.9	7.9	7.7	7.5	7.0	
5	5.7	5.7	5.7	5.6	5.4	4.6	
6	4.2	4.2	4.2	4.2	4.0	3.4	
8	2.4	2.4	2.4	2.4	2.2	1.9	
10	1.3	1.3	1.3	1.3	1.2	0.9	0.2

TABLE VI

Nuclear data for levels in O^{16}

$C^{12}(\alpha,\gamma)O^{16}$ $Q = 7.16$ Mev

$E'_m = 0.2\, T_9^{2/3}$

E^*	7.12	9.58	9.84
E_r	-0.041	2.42	2.68
$J\pi$	1 -	1 -	2 +
Γ_γ (ev)	$(6.6 \pm 2.2) \times 10^{-2}$	6×10^{-3} (within a factor 2)	$(2 \pm 1) \times 10^{-2}$
Γ_α (ev)		$(0.65 \pm 0.03) \times 10^{6}$	$(7.5 \pm 4) \times 10^{2}$
τ (Mev)	3.7×10^{5}	2.1×10^{6}	2×10^{4}
S' (Mev barns)	$\frac{4.7 \times 10^{-3}}{(E'_m + 0.041)^2}$	$\frac{8.2 \times 10^{-3}}{(E'_m - 2.42)^2} + 0.1$	$\frac{4.3 \times 10^{-4}}{(E'_m - 2.68)^2}$
uncertainty in S'	20	≈ 2	≈ 3
uncertainty in $p_r/\rho x_\alpha$		≈ 2	≈ 2
$\omega \frac{\Gamma_\alpha \Gamma_\gamma}{\Gamma}$		1.8×10^{-2}	0.1 ± 0.05

TABLE VII

Rate for C^{12} (α,γ) O^{16}

T	$\log P/\rho x_{\alpha}$	Type
1.0	-20.52	
1.2	-18.88	
1.4	-17.61	
1.6	-16.54	
1.8	-15.65	
2.0	-14.88	
2.5	-13.36	7.12 NR uncertainty ~ 20
3	-12.21	
4	-10.56	
5	- 9.42	
6	- 8.53	
8	- 7.27	
10	- 6.40	
15	- 4.81	7.12 and 9.58 uncertainty ~5
20	- 3.73	
30	- 2.03	
40	- 1.27	9.58 and 9.84 uncertainty ~2
50	- 0.76	

TABLE VIII

Nuclear data for levels in Ne^{20}

O^{16} (α,γ) Ne^{20} Q = 4.73 Mev. $E_m = 0.25\ T_8^{2/3}$

E^*	5.64	5.80	6.75
E_r	0.91	1.07	2.02
$J\pi$	3 −	1 −	0 +
$\omega \frac{\Gamma_\alpha \Gamma_\gamma}{\Gamma}$ (ev)	$(3 \pm 2) \times 10^{-3}$	3×10^{-2}	
Γ_γ/Γ	$\begin{Bmatrix} 0.07 \pm 0.01 \\ 0.3 \pm 0.2 \end{Bmatrix}$	<0.006	
Γ_γ	$(4 \pm 3) \times 10^{-4}$	10^{-2}	
Γ_α (ev)	$(6 \pm 4) \times 10^{-3}$	2	19000
$\underline{\chi}$ (Mev)	10^5	$10^{6.3}$	$10^{7.6}$
$S^\bullet$ (Mev-barns)	$\frac{7 \times 10^{-5}}{(0.9 - E^{\bullet}_m)^2}$	$\frac{1.3 \times 10^{-2}}{(1.1 - E'_m)^2}$	$\frac{0.8}{(2.0 - E'_m)^2}$
S (average) $1 < T < 2$	2×10^{-4}	2.5×10^{-2}	3×10^{-1}
Resonant $P_r/\rho x_\alpha$	$\frac{6.3 \times 10^2 \times 10^{-45.9/T}}{T^{3/2}}$	$\frac{6.3 \times 10^3 \times 10^{-54.4/T}}{T^{3/2}}$	$\frac{2.1 \times 10^4 \times 10^{-102/T}}{T^{3/2}}$
uncertainty in $P_r/\rho x_\alpha$	≈ 2	≈ 10	≈ 10

TABLE IX

Rate for O^{16} (α,γ) Ne^{20}

T	log $p/\rho x_\alpha$	Type
1.0	-27.68	5.81 and 6.75 nonresonant uncertainty ~10
1.5	-23.12	
2.0	-19.4	
2.5	-16.13	5.64 resonant uncertainty ~2
3.0	-13.19	
5.0	- 7.41	
6.0	- 5.84	
8.0	- 4.0	5.64 and 5.81 resonant uncertainty ~10
10	- 2.87	
20	- 0.84	
30	+ 0.5	6.75 and higher levels resonant uncertainty < 10
40	1.2	
50	1.41	

TABLE X

Parameters of key levels in O^{17} for the reaction

$C^{13} + d \rightarrow O^{17*} \rightarrow O^{16} + n$

($Q = 2.2145 \pm 0.0014$ Mev)

E*	Jπ	Γ Kev	θ_α^2	Γ_α Mev	uncertainty in Γ_α	$\omega\Gamma_\alpha\Gamma_n/\Gamma$ Mev	$\gamma^2 \xi_\ell'^2$ Mev	uncertainty in $\gamma^2 \xi_\ell'^2$	$\omega\Gamma_n$ Kev	$\{(E'_m - E_r)^2 + \Gamma^2/4\}$ XS'_{BW}	Er Mev
6.37	1/2+	130		1×10^{-74}	5	1×10^{-74}	1.4×10^5	5	130	4×10^3 (uncertainty 5)	0.013
6.869				2×10^{-8}	15	2×10^{-8}	1×10^5	50	1	20	0.512
7.170	5/2	2.7	(0.5)*	2×10^{-6}*	5	6×10^{-6}	4×10^4	5	8	70	0.813
7.28	3/2+	210		2.6×10^{-5}	5	5×10^{-5}	1.4×10^5	5	420	1×10^4 (uncertainty 5)	0.92
7.95	1/2	90	0.09*	8×10^{-3}*	2	8×10^{-3}					1.59
8.08	3/2	75	0.11*	6.8×10^{-3}*	2	1.2×10^{-2}					1.72

* See Walton, Clement and Boreli (1957)

TABLE XI

Values of the rate of the reaction

$C^{13}(\alpha,n)O^{16}$

T	log $p/\rho X_\alpha$	Contribution
0.6	-18.8	
0.8	-15.2	
1.0	-13.2	
1.1	-12.2	
1.2	-11.4	
1.5	-10.7	Nonresonant Uncertainty $\simeq$ 3
2	- 7.0	
3	- 4.1	
4	- 2.2	
5	- 0.9	
6	+ 0.1	
8	+ 1.5	

TABLE XII

Parameters of Key levels in F^{18} for the reactions

$N^{14}(\alpha,\gamma)F^{18}$ (Q = 4.404 ± 0.004 Mev)

$N^{14}(\alpha,p)O^{17}$ (Q = -1.193 ± 0.001 Mev)

E*	Er	J_{π}	Γ Kev	Γ_α Mev	θ_α^2	$\frac{\omega\Gamma_\alpha\Gamma_\gamma}{\Gamma}$ Mev	Nonresonant contribution to $N^{14}(\alpha,\gamma)F^{18}$: $\gamma^*\xi_\ell^{12}$	$\{(E'_m-Er)^2+\Gamma^2/4\}$ x S'_{BW}	uncertainty in S'_{BW}
4.355	-0.049	(0,1,2)-					1.6 x 10^6	1	15
4.651	0.247			1 x 10^{-16}		1 x 10^{-16}	3 x 10^5	2 x 10^{-1}	75
4.741	0.337			1.3 x 10^{-13}		1.3 x 10^{-13}	3.3 x 10^5	2.2 x 10^{-1}	75
4.844	0.440	+		2.3 x 10^{-11}		2.3 x 10^{-11}	3.5 x 10^5	2.3 x 10^{-1}	75
4.964	0.560	(0,1,2)-		6 x 10^{-9}		4 x 10^{-9}	1.9 x 10^6	1.2	15
							Erp	$\frac{\omega\Gamma_\alpha\Gamma_p}{\Gamma}$	uncertainty in $\frac{\omega\Gamma_\alpha\Gamma_p}{\Gamma}$
5.295	0.891			1x 10^{-6}		7.5 x 10^{-7}			
6.800	2.396	2-	79 ± 5*	6.4 x 10^{-2*}	0.18	5 x 10^{-6}	1.203	1.45 x 10^{-2*}	2 x 10^{-2}	20%

(Erp designates the energy of the level with reference to O^{17} + p.)

*See Brown (1962)

TABLE XIII

$N^{14} + \alpha$

Values of the Reaction Rate

T	$\log p/\rho x_\alpha$	Contribution
$10^8\,°K$		
0.2	-45.00	Nonresonant E*: 4.355 Uncertainty ≈ 20
0.3	-38.00	
0.4	-33.50	
0.5	-29.10	Resonant E*: 4.651 Uncertainty ≈ 25
0.6	-25.05	
0.8	-20.05	
1.0	-17.10	
1.1	-16.00	
1.2	-15.07	Resonant: 4.651, 4.741, 4.844, Uncertainty ≈ 20
1.5	-12.84	
2	-10.28	
3	- 6.91	Resonant: 4.741, 4.844, 4.964, 5.295 Uncertainty ≈ 10
4	- 4.92	
5	- 3.62	
6	- 2.66	
8	- 1.43	
10	- 0.47	
20	2.40	$N^{14}(\alpha,p)\,O^{17}$ - many levels, especially 6.800 Uncertainty ≈ 2
30	3.88	
40	4.66	
50	5.18	

TABLE XIV

Parameters of key levels in F^{19} for the reaction

$N^{15} + \alpha \rightarrow F^{19} + \gamma$ ($Q = 4.012 \pm 0.001$ Mev)

E* Mev	Er Mev	J_π	Γ Kev	Γ_α Mev	θ_α^2 to 10%	$\omega \frac{\Gamma_\alpha \Gamma_\gamma}{\Gamma}$ Mev	$\gamma^* \xi_\alpha'^2$ Mev	$\omega \Gamma_\gamma$ ev	$\{(E_m-E_r)^2+\Gamma^2\}^{\frac{1}{2}}$ x S'	uncertainty in S'
4.036*	0.024			3×10^{-64}		1.2×10^{-63}	3×10^5	8	0.5	200
4.385+	0.373			8×10^{-13}		1.6×10^{-12}	3.5×10^5	8	0.58	150
4.563+	0.551			1×10^{-9}		4×10^{-9}	3.8×10^5	8	0.64	150
4.760*	0.748			1×10^{-7}		4×10^{-7}	4×10^5	8	0.70	120
5.102+	1.090			1.3×10^{-5}		8×10^{-6}	5.3×10^5	8	0.88	100
5.339*	1.327	½(3/2-)	< 2	$(\Gamma_\gamma < \Gamma_\alpha)$		6×10^{-6}	2.3×10^6	6	3.8	50
5.481+	1.469	3/2+*	3.2*		0.2*	8×10^{-6}				

*See Smotrich, Jones, McDermott and Benenson (1961) and

+See Silbert and Jarmie (1961)

*See Ajzenberg-Selove and Lauritzen

TABLE XV

$N^{15}(\alpha,\gamma)\ F^{19}$

Values of the rate of the reaction

T	log $p/\rho x_{\alpha}$	Contribution
0.2	−44.85	
0.3	−37.92	Nonresonant
0.4	−33.47	E*: 4.036
0.5	−30.33	Uncertainty ≈ 200
0.6	−27.92	
0.8	−23.82	
1.0	−19.26	
1.1	−17.55	Resonant
1.2	−16.13	E*: 4.385
1.5	−13.00	Uncertainty ≈ 50
2	− 9.82	
3	− 6.17	
4	− 3.85	Resonant
5	− 2.30	4.385, 4.563, 4.760
6	− 1.18	Uncertainty ≈ 25
8	0.34	
10	1.32	
20	3.78	Resonant: numerous levels
30	4.81	5.102, 5.339, 5.481
40	5.38	Uncertainty ≈ 5
50	5.75	

FIGURE CAPTIONS

Fig. 1. Graphical display of energy relations for $3\alpha \rightarrow C^{12}$ (not drawn to scale).

Fig. 2. Range of excitation in C^{12} at various temperatures for $\alpha + Be^{8} \rightarrow C^{12} + \gamma$.

Fig. 3. Mean life in sec. of an He^{4} nucleus and rate of energy production for $3\alpha \rightarrow C^{12}$ at various α particle densities in gm/cm^{3}.

Fig. 4. Range of excitation in O^{16} at various temperatures for $C^{12}(\alpha,\gamma)O^{16}$.

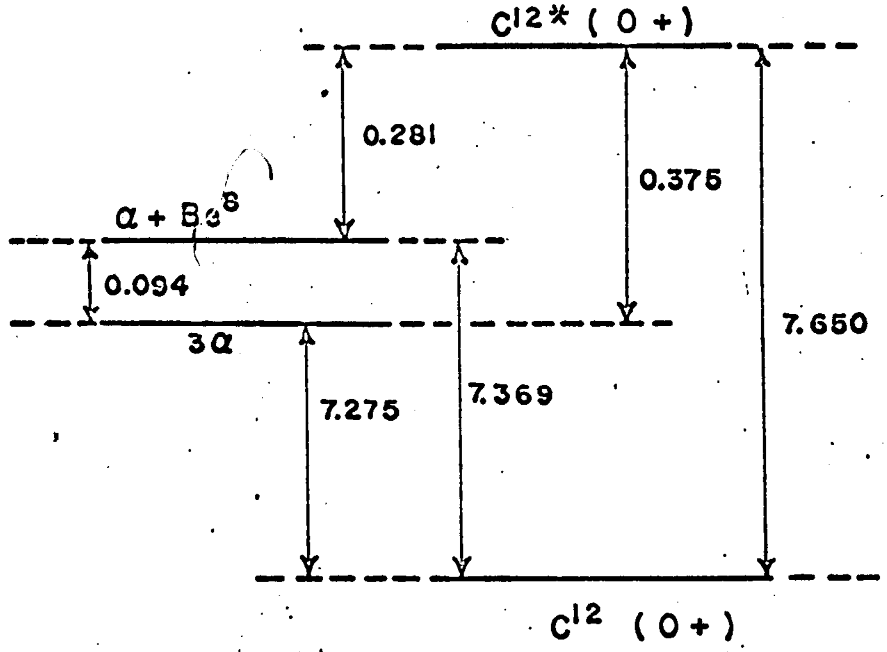
Fig. 5. Range of excitation in Ne^{20} at various temperatures for $O^{16}(\alpha,\gamma)Ne^{20}$.

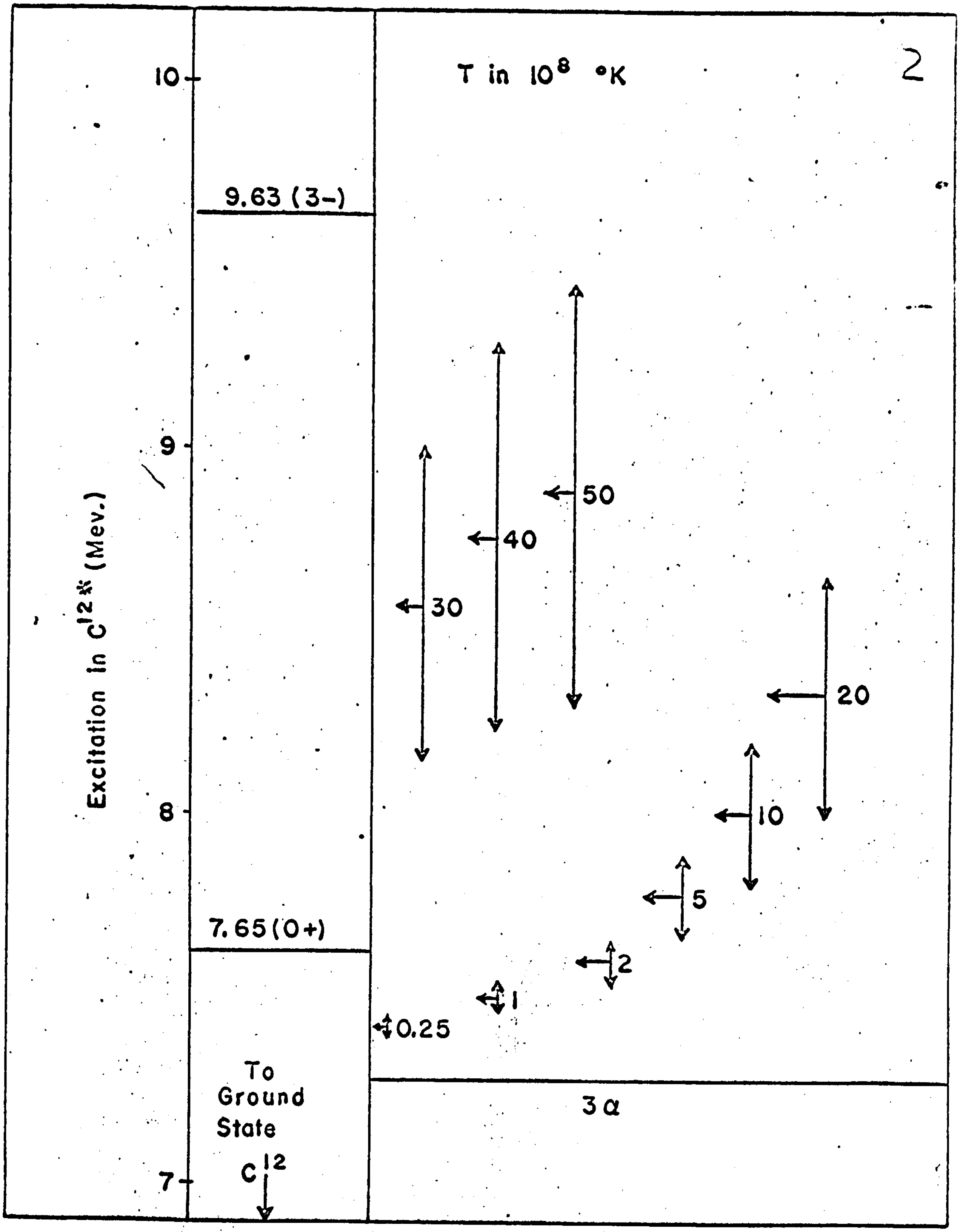
Fig. 6. Range of excitation in O^{17} at various temperatures for $C^{13}(\alpha,n)O^{16}$.

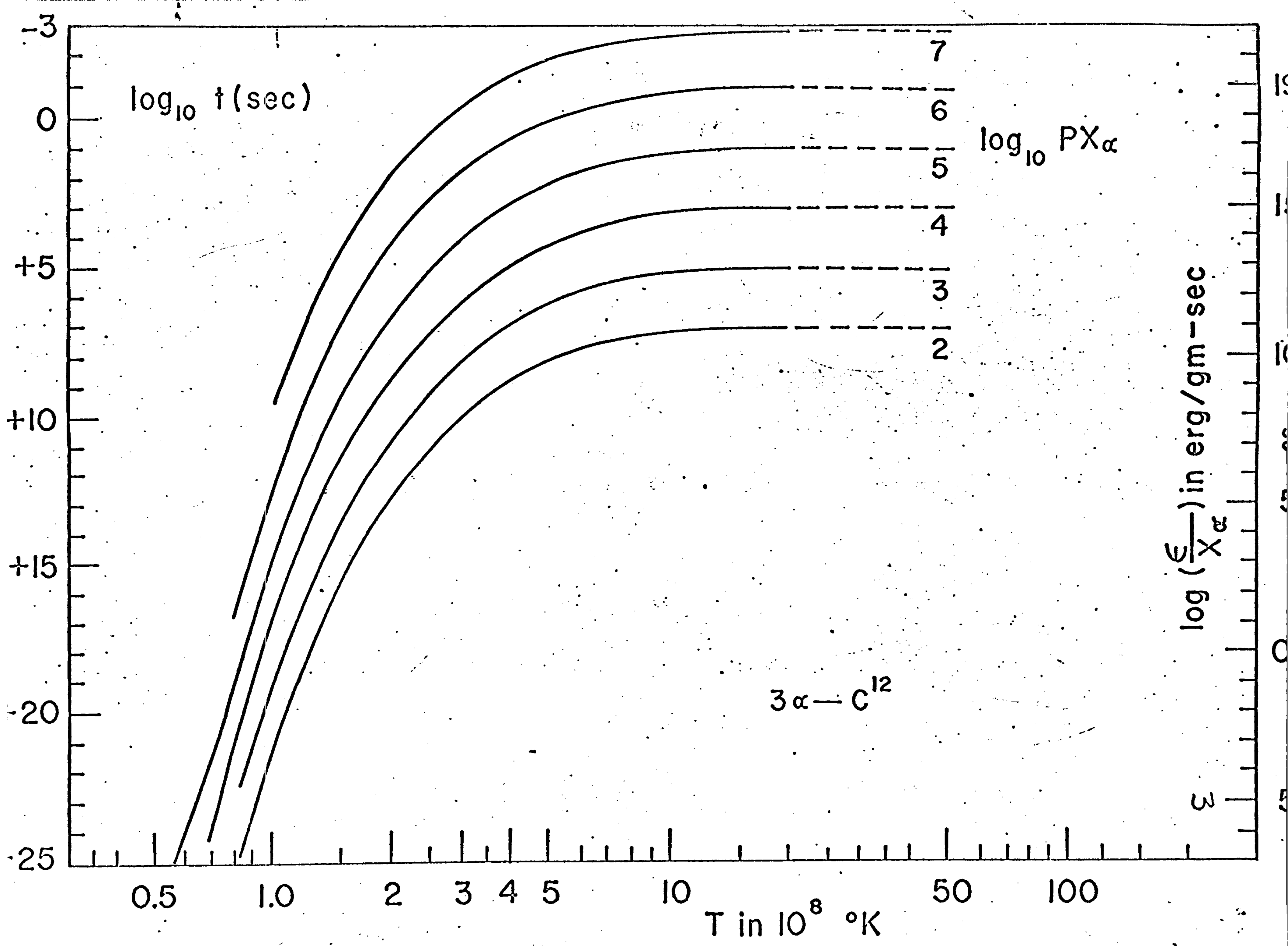
Fig. 7. Range of excitation in F^{18} at various temperatures for $N^{14} + \alpha$.

Fig. 8. Range of excitation in F^{19} at various temperatures for $N^{15}(\alpha,\gamma)F^{19}$.

Fig. 9. Values of log $q\rho$ for ~~$C^{13}(\alpha,n)O^{16}$,~~ $C^{12}(\alpha,\gamma)O^{16}$, and $O^{16}(\alpha,\gamma)Ne^{20}$.

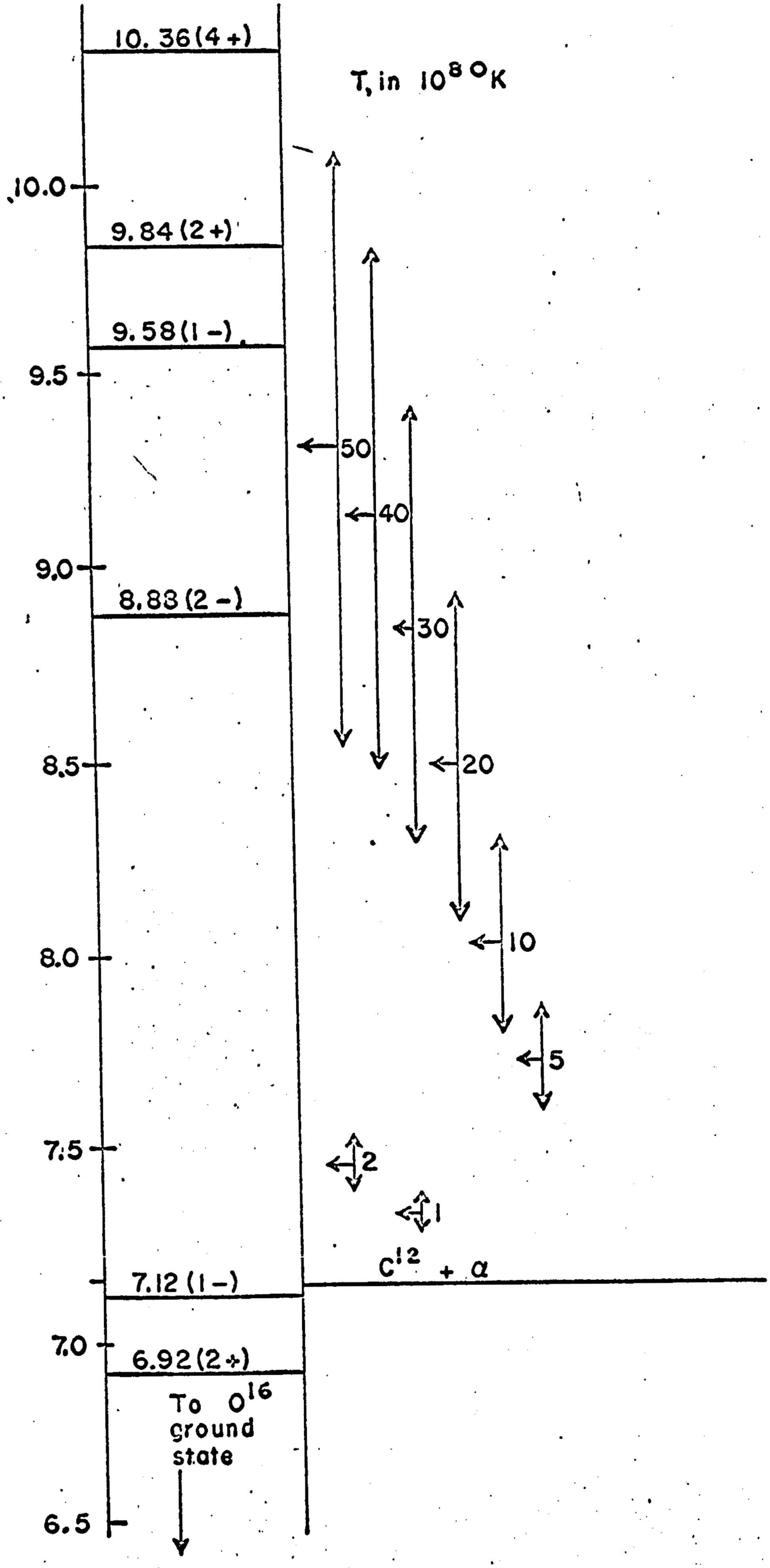

C12* (0 +)
0.281
0.375
α + Be8
0.094
3α
7.275
7.369
7.650
C12 (0 +)


2
T in 10^8 °K
10
9
8
7
Excitation in C^{12*} (Mev.)
9.63 (3-)
7.65 (0+)
To
Ground
State
C^{12}
50
40
30
20
10
5
2
1
0.25
3α

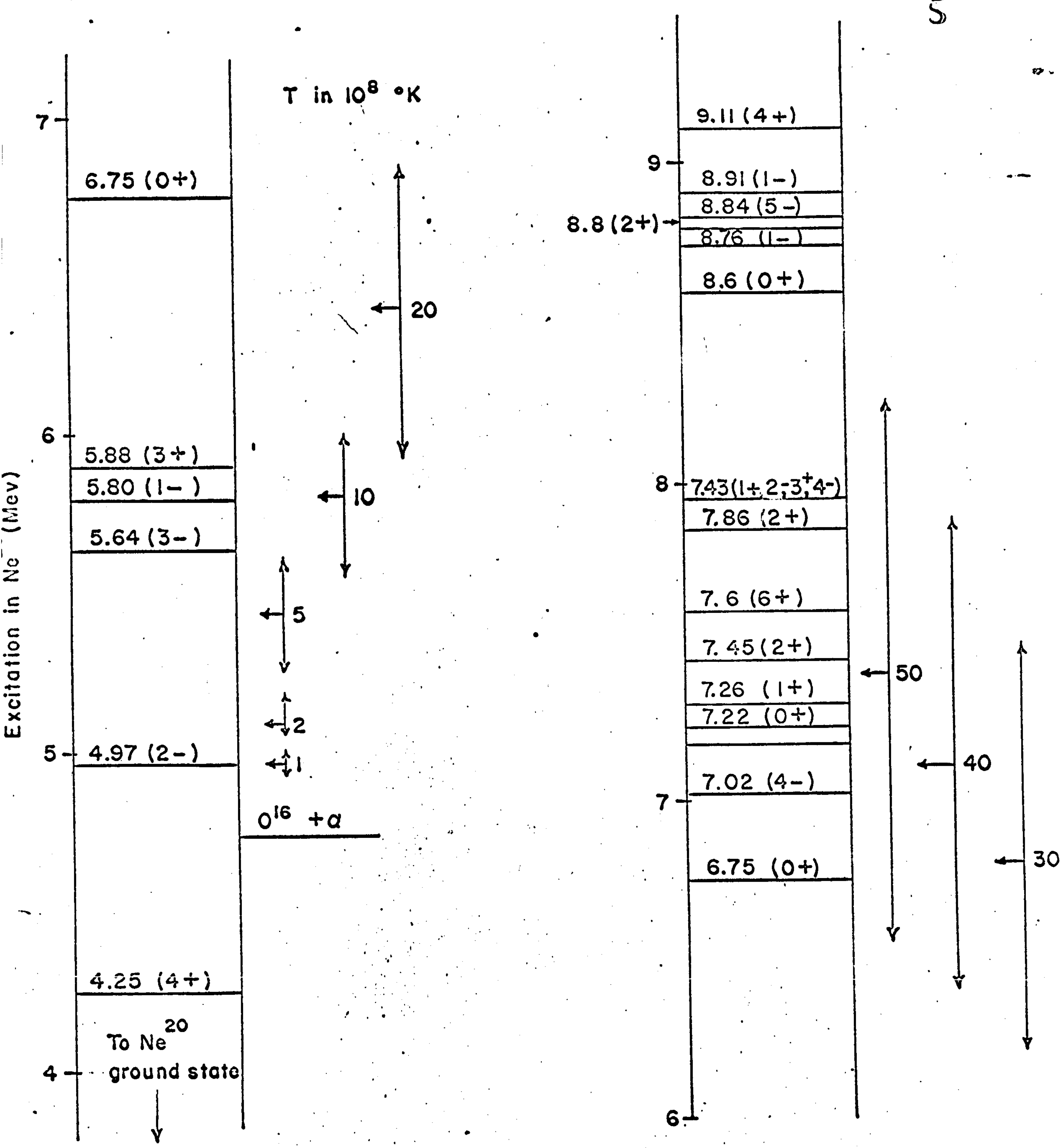


log₁₀ t (sec)
log₁₀ PXα
7
6
5
4
3
2
3α — C¹²
T in 10⁸ °K
log (ε/Xα) in erg/gm-sec
-3
0
+5
+10
+15
-20
-25
0.5
1.0
2
3
4
5
10
50
100
3

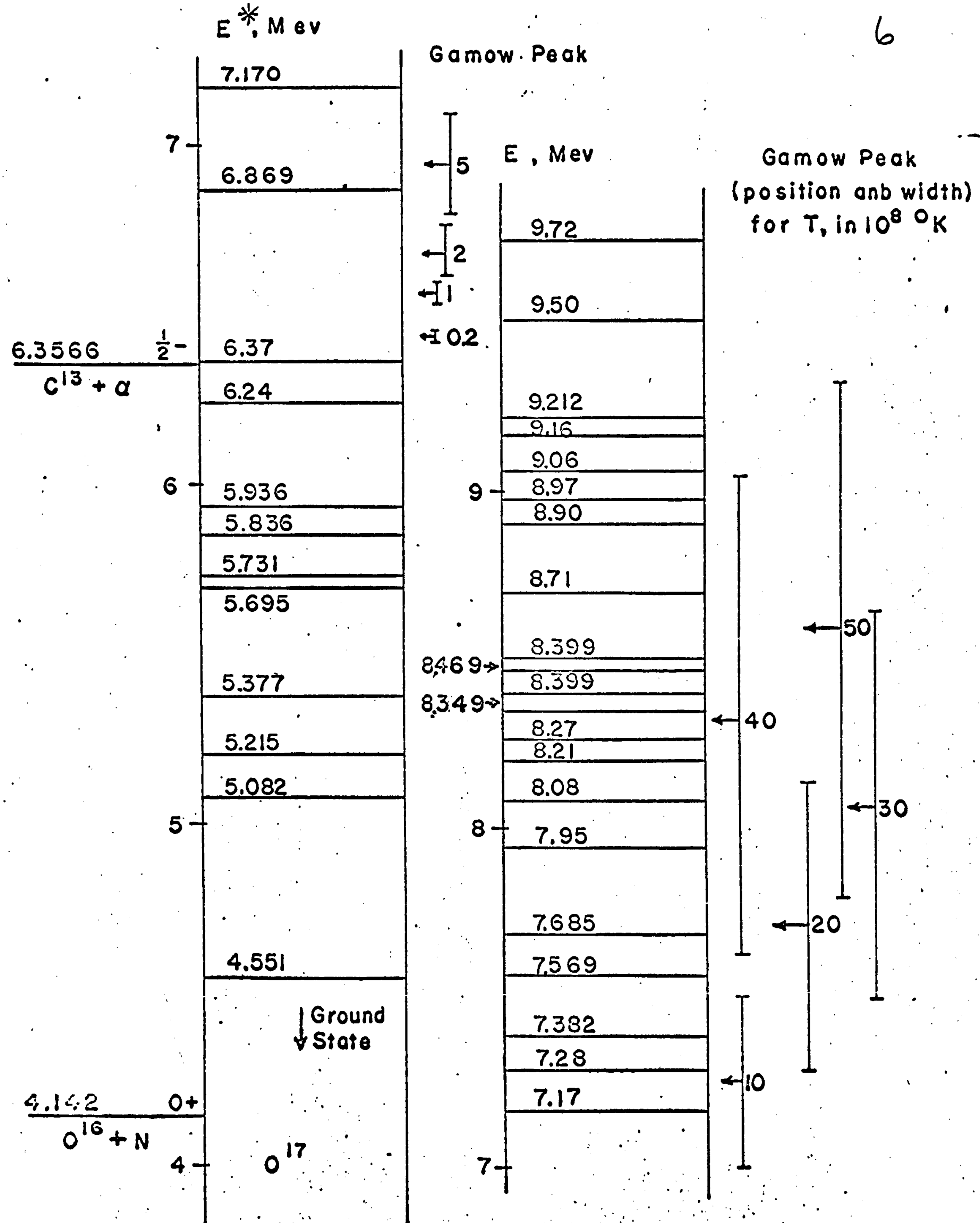
4


10. 36(4+)
T, in 10^8 °K
10.0
9.84 (2+)
9.58 (1-)
9.5
50
40
9.0
8.83 (2-)
30
8.5
20
8.0
10
5
7.5
2
1
C^{12} + α
7.12 (1-)
7.0
6.92 (2+)
To O^{16}
ground
state
6.5

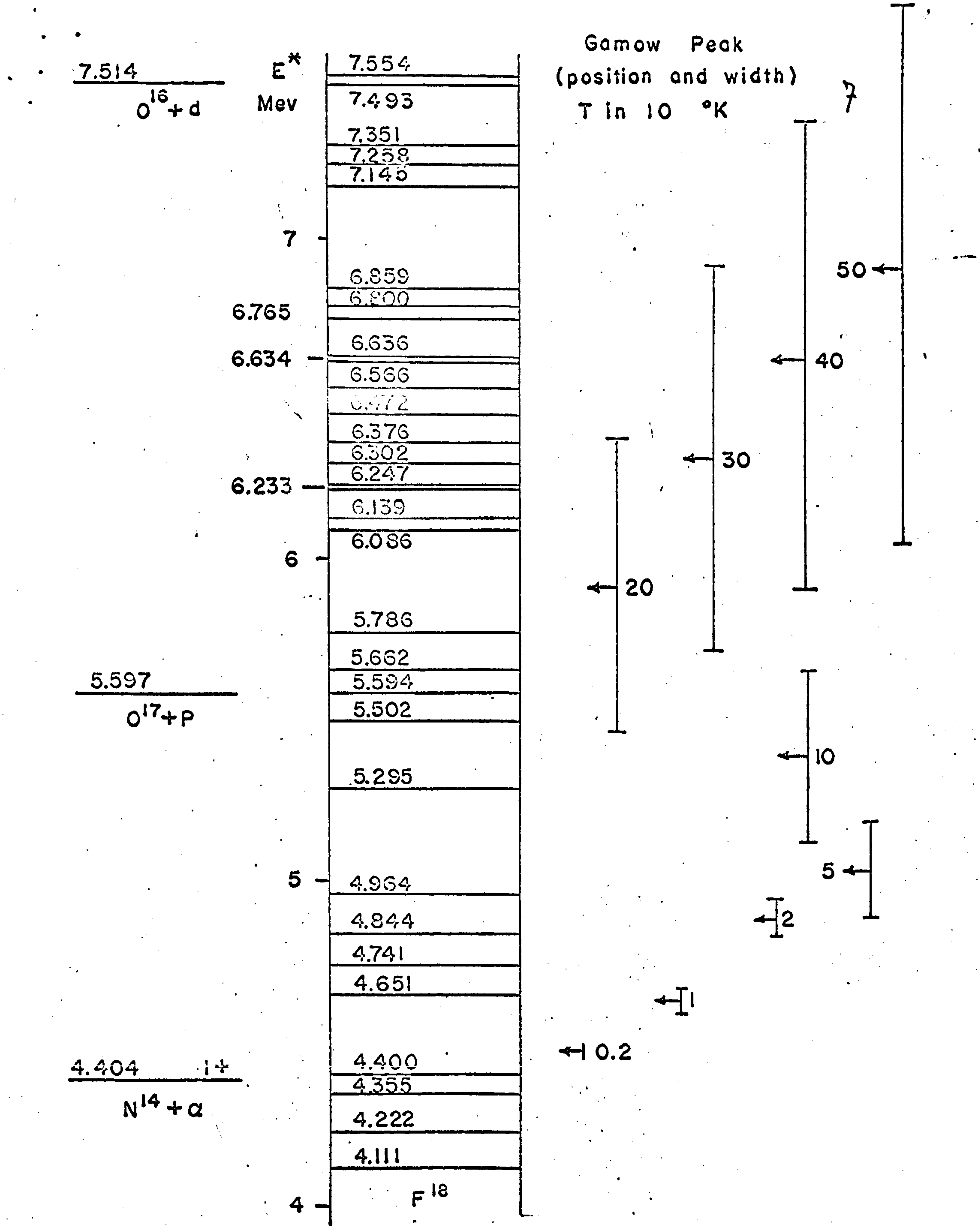
5

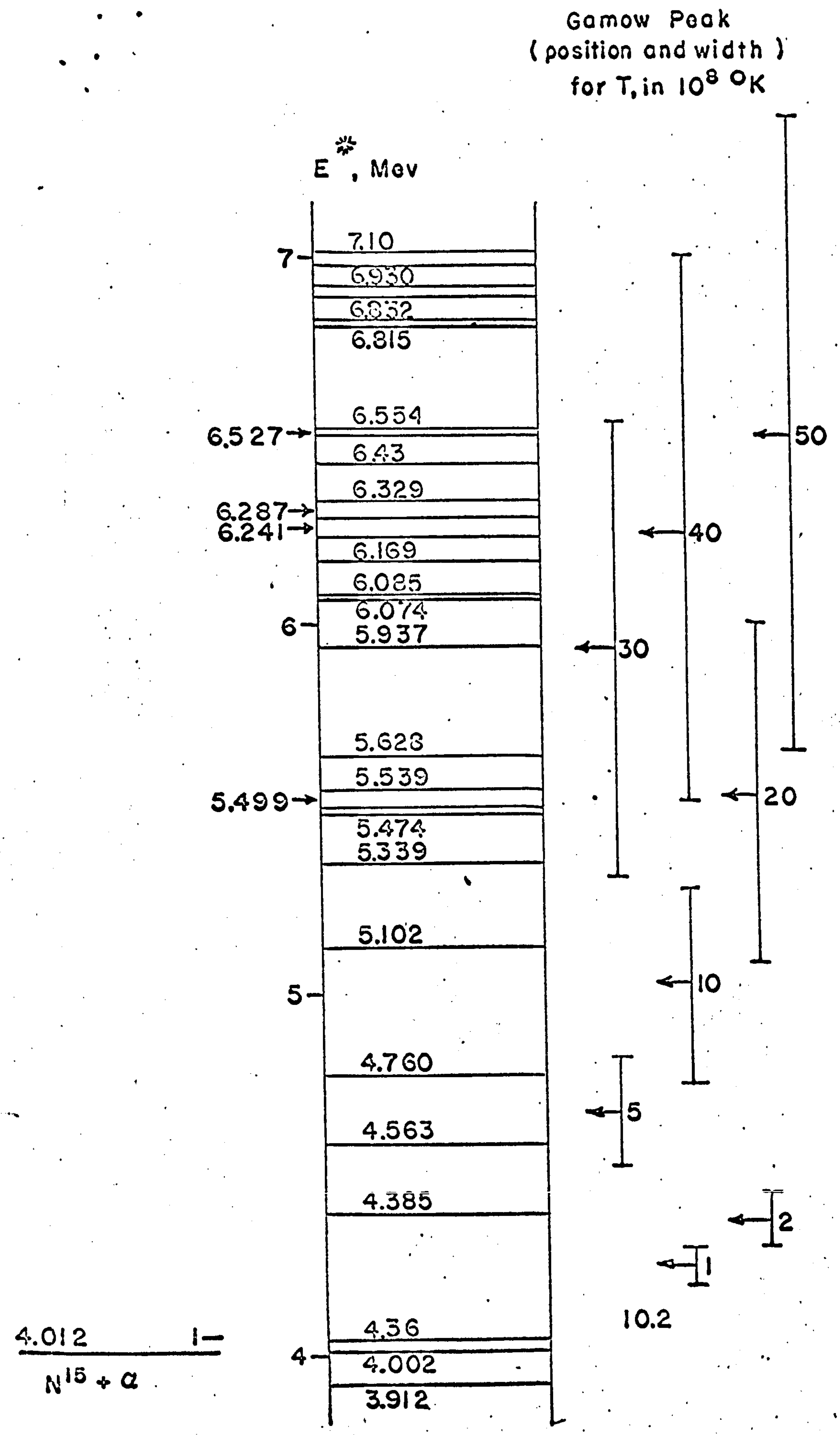

T in 10^8 °K
Excitation in Ne^{20} (Mev)
7
6.75 (0+)
20
6
5.88 (3+)
5.80 (1-)
10
5.64 (3-)
5
2
5
4.97 (2-)
1
O^{16} + α
4.25 (4+)
To Ne^{20} ground state
4
9.11 (4+)
9
8.91 (1-)
8.84 (5-)
8.8 (2+)
8.76 (1-)
8.6 (0+)
8
7.93 (1+, 2-, 3+, 4-)
7.86 (2+)
7.6 (6+)
7.45 (2+)
50
7.26 (1+)
7.22 (0+)
40
7.02 (4-)
7
6.75 (0+)
30
6

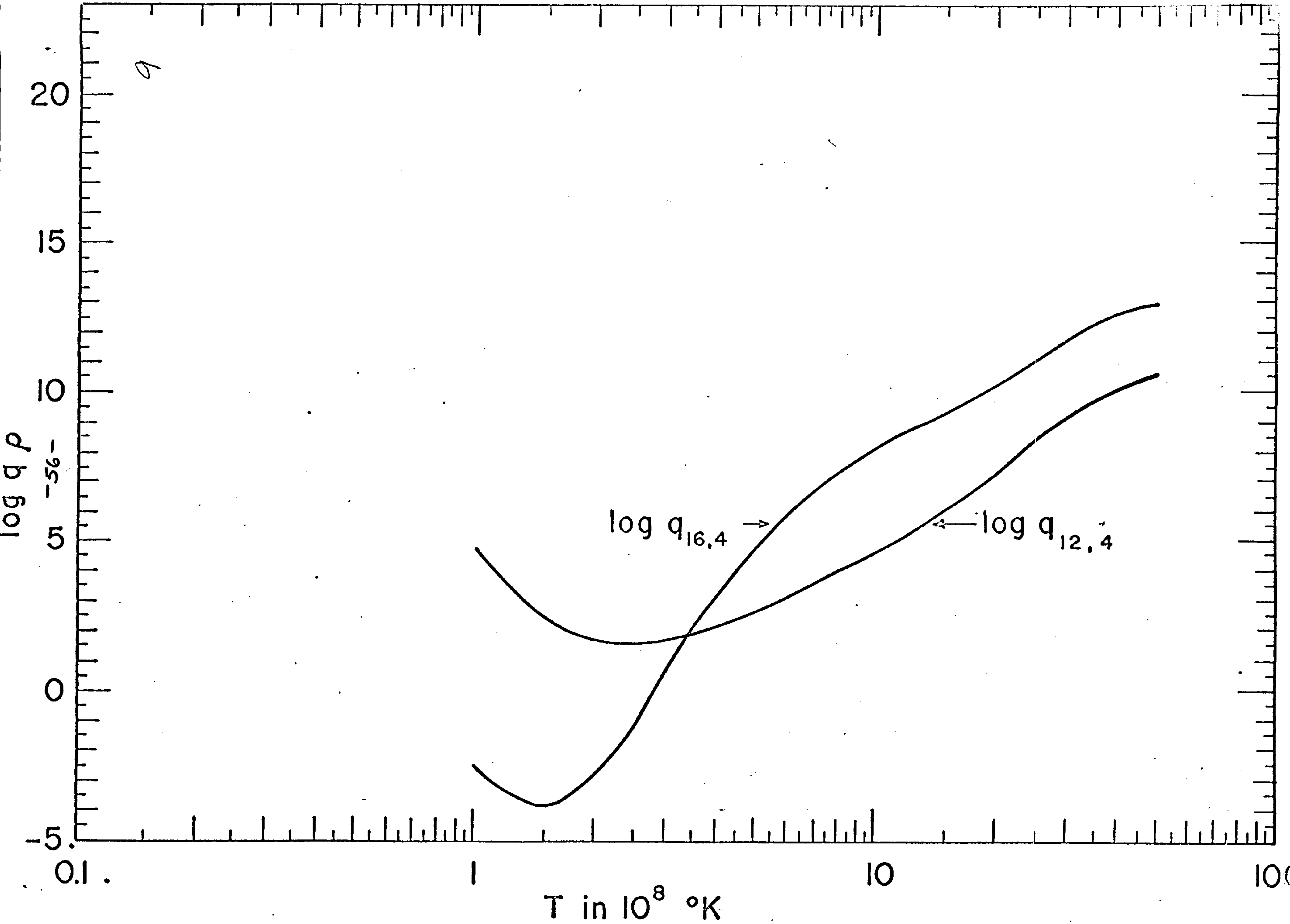
6



E*, Mev
Gamow Peak
7.170
7
6.869
5
2
1
0.2
6.3566
½-
6.37
C13 + α
6.24
6
5.936
5.836
5.731
5.695
5.377
5.215
5.082
5
4.551
Ground State
4.142
0+
O16 + N
4
O17
E, Mev
9.72
9.50
9.212
9.16
9.06
9
8.97
8.90
8.71
8.469
8.399
8.399
8.349
8.27
8.21
8.08
8
7.95
7.685
7.569
7.382
7.28
7.17
7
Gamow Peak (position anb width) for T, in 10^8 °K
50
40
30
20
10


7.514
$O^{16}+d$
E^*
Mev
7.554
7.493
7.351
7.258
7.145
7
6.859
6.800
6.765
6.636
6.634
6.566
6.472
6.376
6.302
6.247
6.233
6.139
6.086
6
5.786
5.662
5.597
5.594
$O^{17}+p$
5.502
5.295
5
4.964
4.844
4.741
4.651
4.404 1+
4.400
$N^{14}+\alpha$
4.355
4.222
4.111
4
F^{18}
Gamow Peak
(position and width)
T in 10 °K
7
50
40
30
20
10
5
2
1
0.2


Gamow Peak
(position and width)
for T, in 10⁸ °K
8
E*, Mev
7.10
6.930
6.852
6.815
6.554
6.527
6.43
6.329
6.287
6.241
6.169
6.085
6.074
5.937
5.623
5.539
5.499
5.474
5.339
5.102
4.760
4.563
4.385
4.36
4.002
3.912
7
6
5
4
50
40
30
20
10
5
2
1
10.2
4.012 1−
N¹⁵ + α


9
log q ρ
20
15
10
5
0
-5.
0.1 .
1
10
100
T in 10⁸ °K
log $q_{16,4}$
log $q_{12,4}$

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